Spatio-Temporal Swarm Planning on Gaussian Probabilistic Roadmaps

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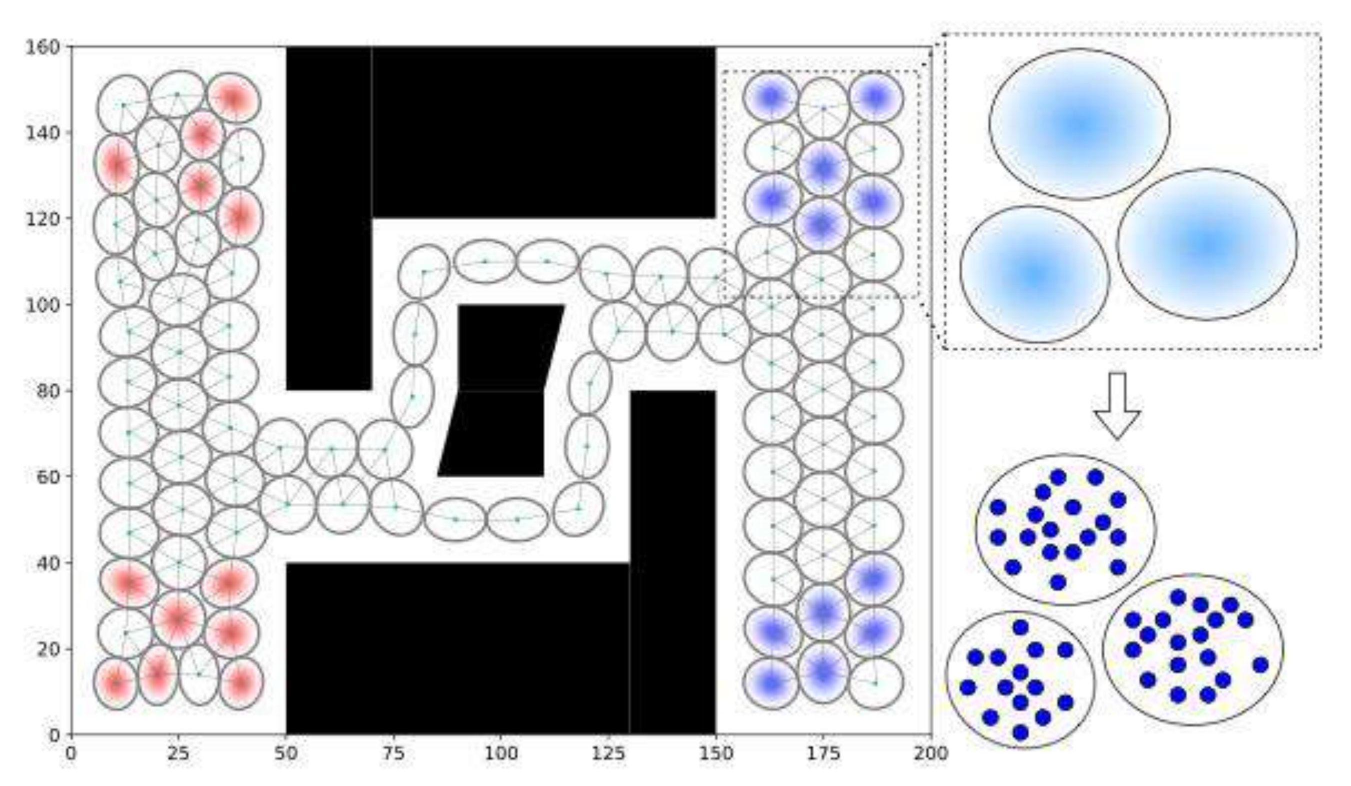
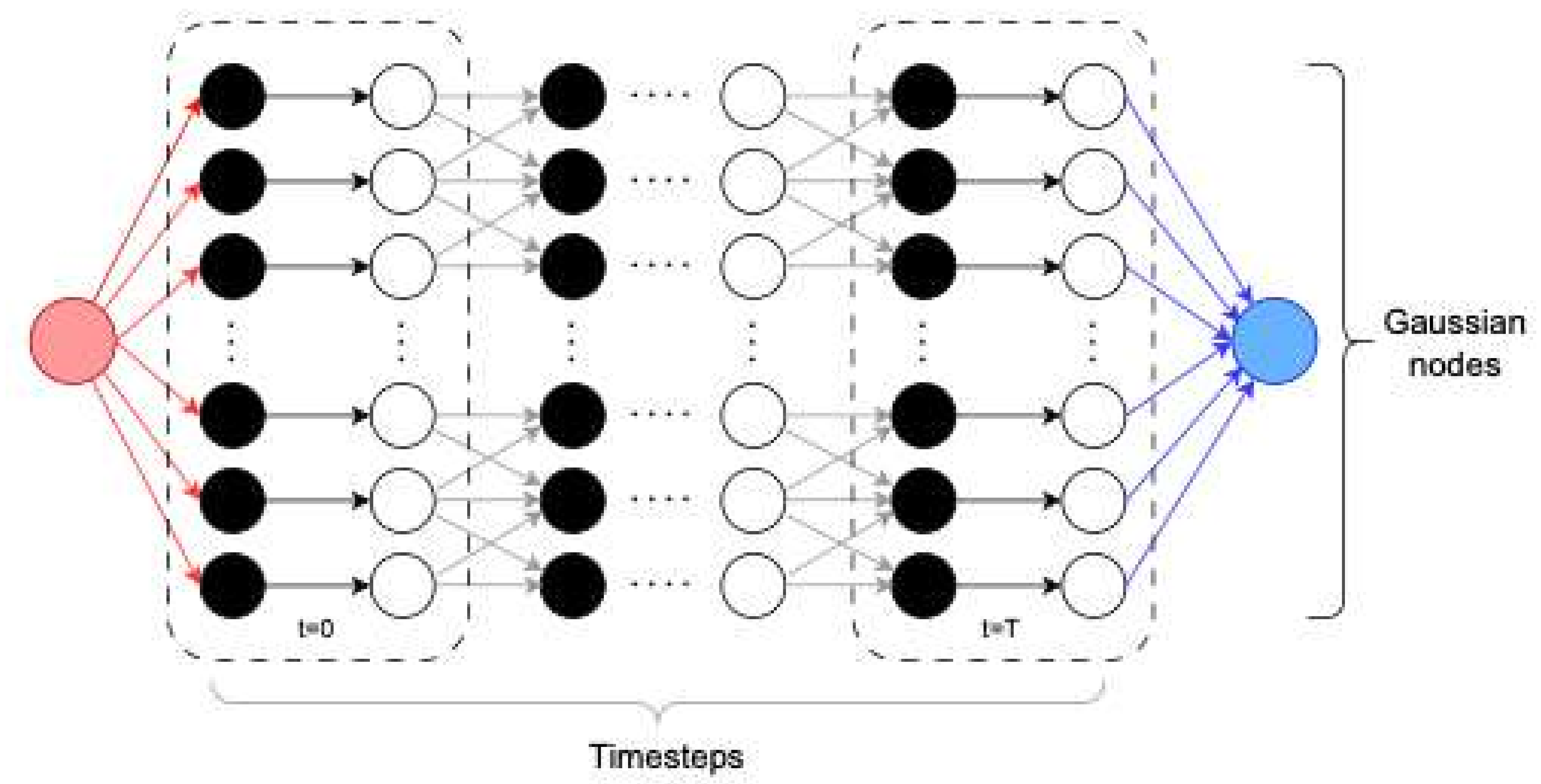
TLDR:
We propose spatio-temporal Gaussian PRM for spatio-temporal swarm planning. By explicitly representing the **capacity constraints** of each Gaussian node, we formulate the planning problem as solving an **min-cost-max-flow** instance on a **time-expanded graph**. This allows for spatio-temporal planning in congested regions and multi-swarm planning on a shared map.

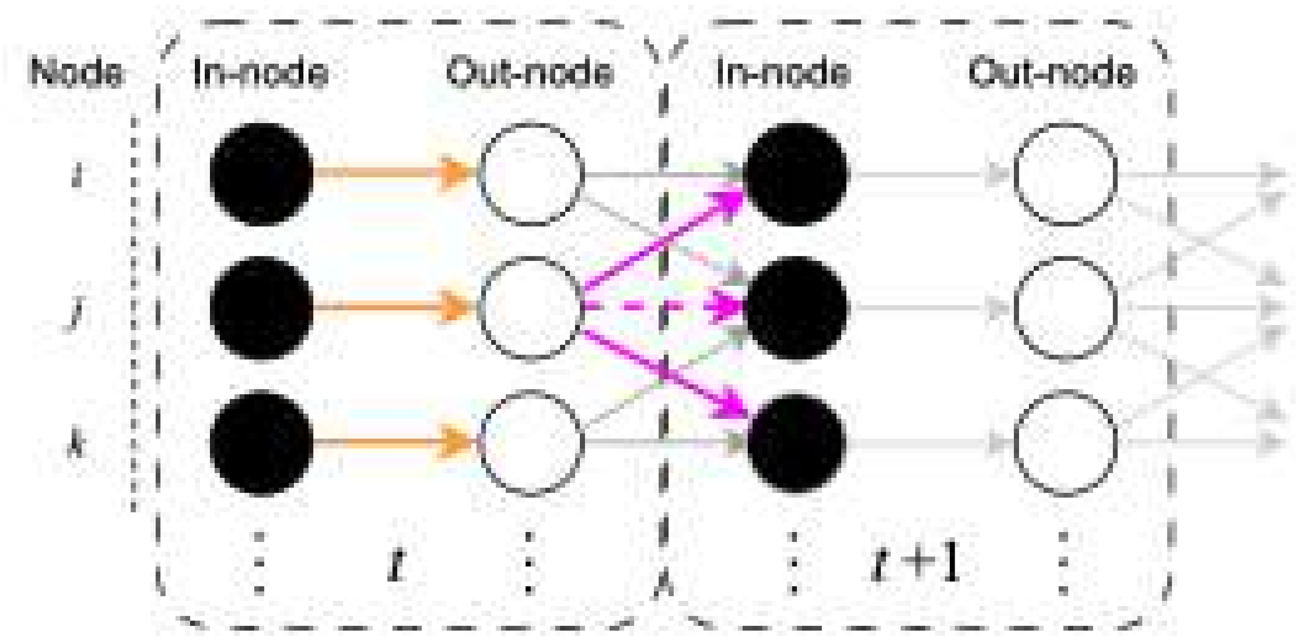
I. Spatio-Temporal Swarm Planning



- Objective: Finding a spatio-temporal plan for a swarm of robots to travel from start to goal configuration and minimize the **sum of distance cost**.
- SwarmPRM [1] proposes to use Gaussian Mixture Models (GMMs) to represent swarm states and solves a linear program to find the min-distance **spatial plan**.
- In congested regions, **temporal coordination** is necessary for finding a feasible plan.

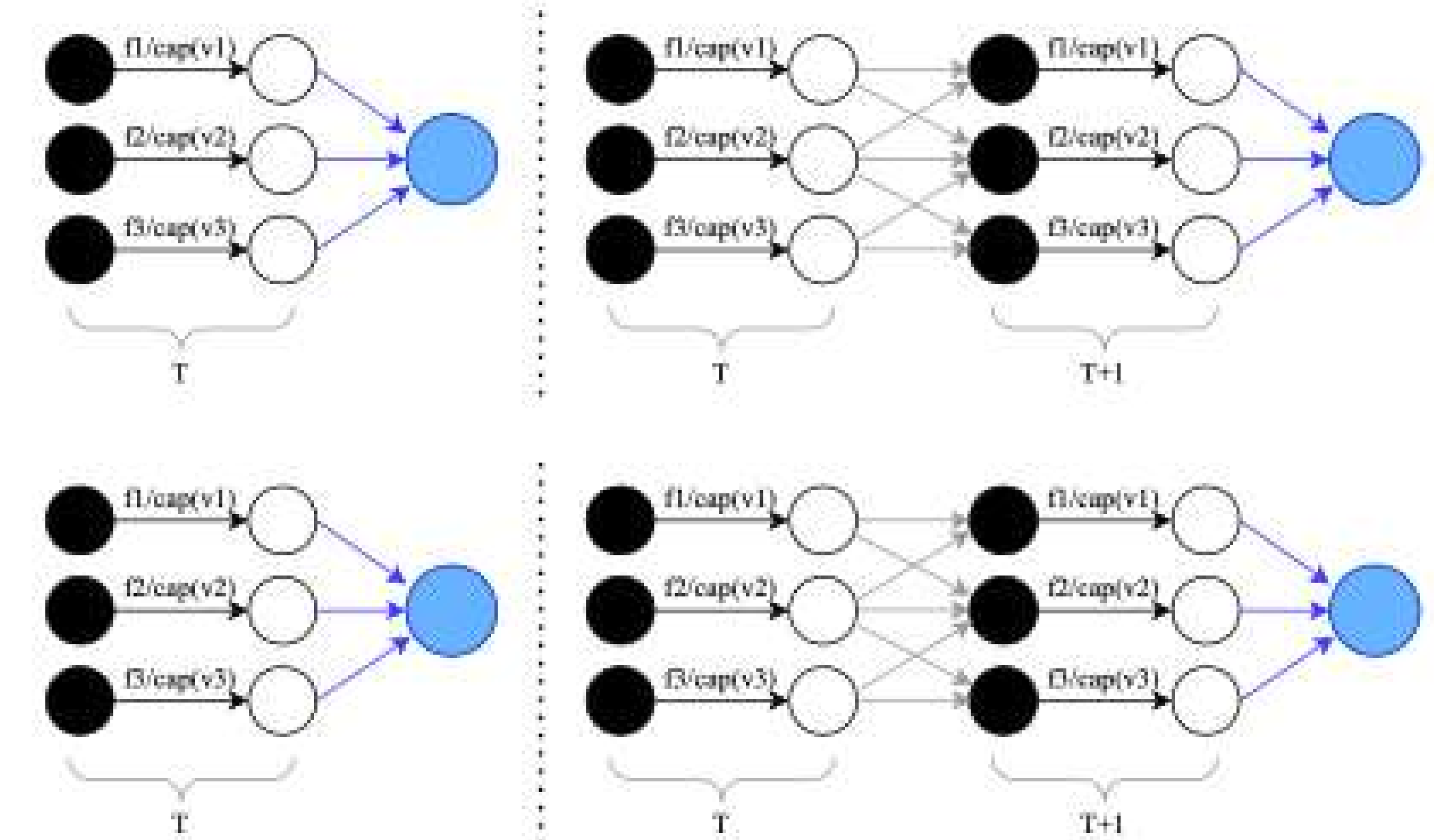
II. Time-expanded Graph (TEG) [2]



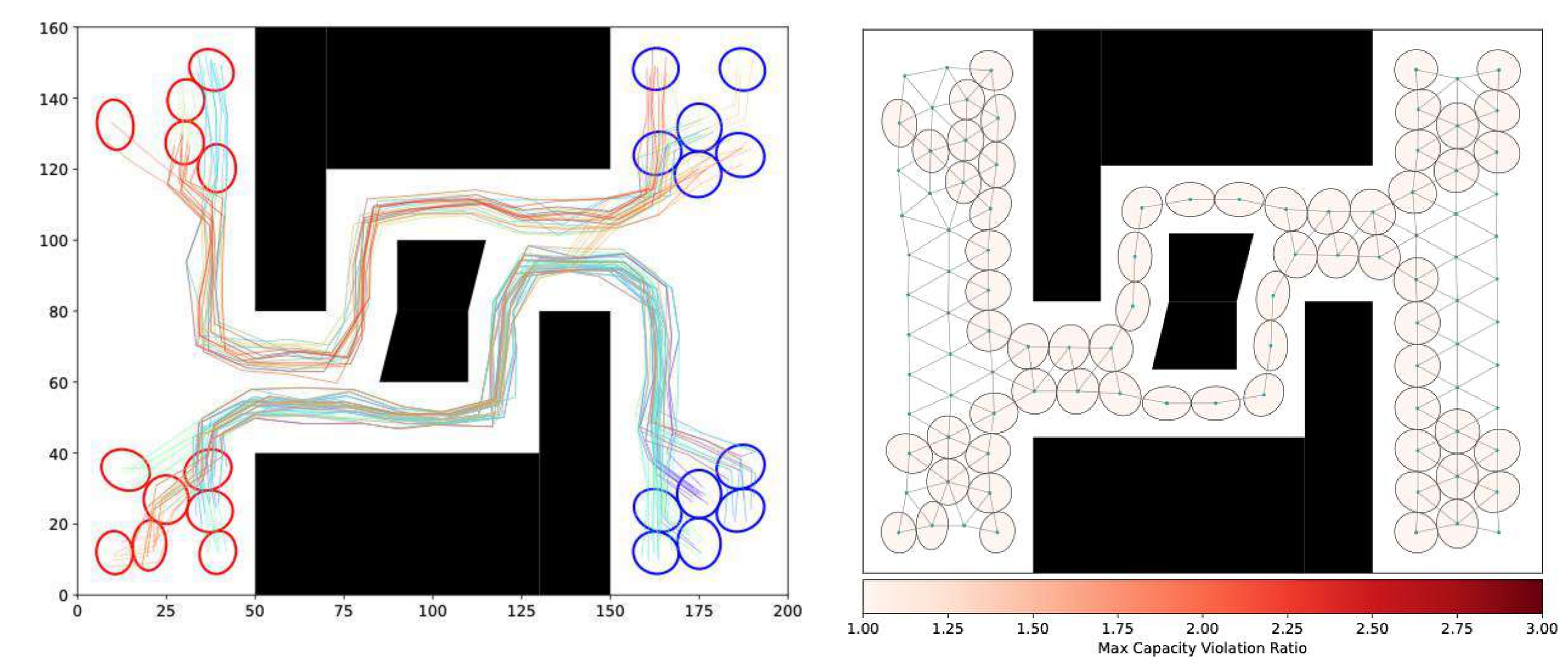


- Edges in the TEG represents possible transitions between consecutive time steps, encoding possible transitions (**move**, **wait**).
- To represent the geometric capacity constraint per node, each graph node is split into an in-node and an out-node, connected by an edge with a capacity constraint.
- A spatio-temporal trajectory for one agent corresponds to one unit of flow on the TEG.
- A network flow with capacity equal to the swarm size corresponds to feasible spatio-temporal plan.

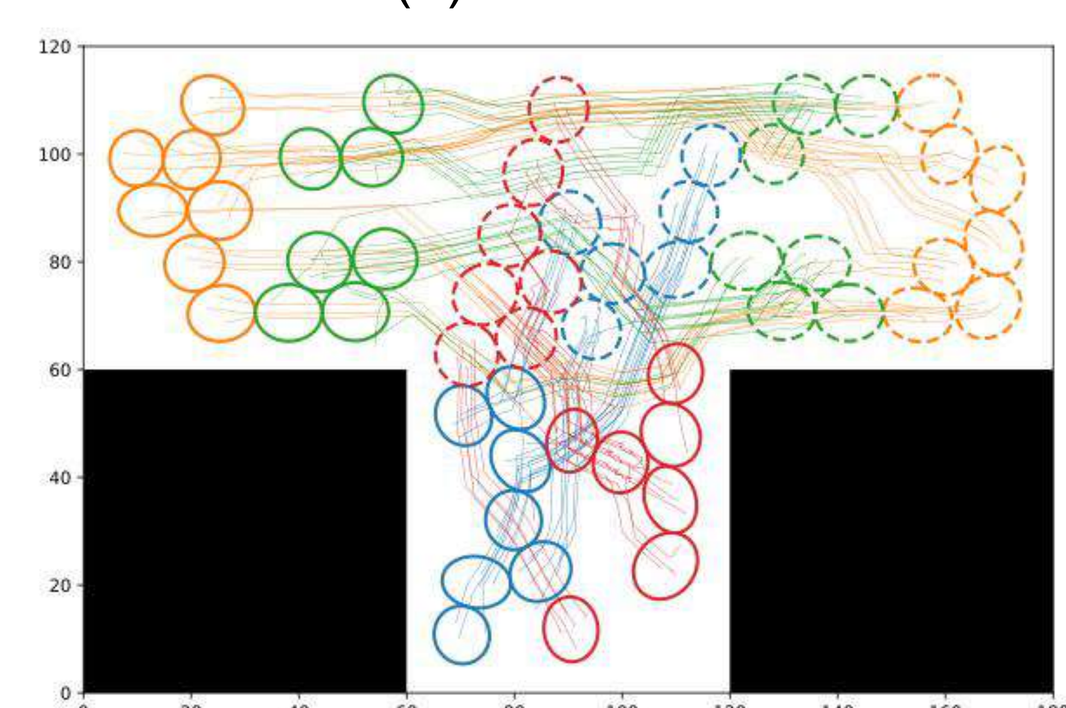
III. Incremental Min-Cost Max-Flow Solving



- Due to the capacity constraints on each node, the maximum flow could be smaller than the swarm size. In that case, we increment the number of time step in the TEG.
- The solution from previous time step can construct a partial solution for the updated TEG. We therefore find the solution incrementally.



(a) (b)



(c)

(a): example solution
(b): capacity violation rate
(c): multi-swarm solution

[1] Hu, Yunze, Xuru Yang, Kangjie Zhou, et al. “SwarmPRM: Probabilistic Roadmap Motion Planning for Large-Scale Swarm Robotic Systems.” arXiv:2402.16699. Preprint, arXiv, October 13, 2024. https://doi.org/10.48550/arXiv.2402.16699.
[2] Yu, Jingjin, and Steven M. LaValle. “Multi-Agent Path Planning and Network Flow.” In *Algorithmic Foundations of Robotics X*, edited by Emilio Frazzoli, Tomas Lozano-Perez, Nicholas Roy, and Daniela Rus. Springer, 2013. https://doi.org/10.1007/978-3-642-36279-8_10.