GHOST: Solving the Traveling Salesman Problem on Graphs of Convex Sets [1]

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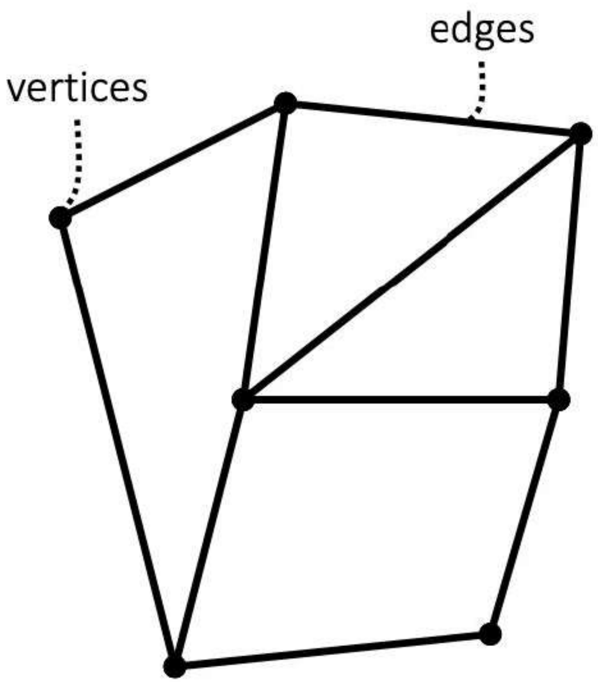
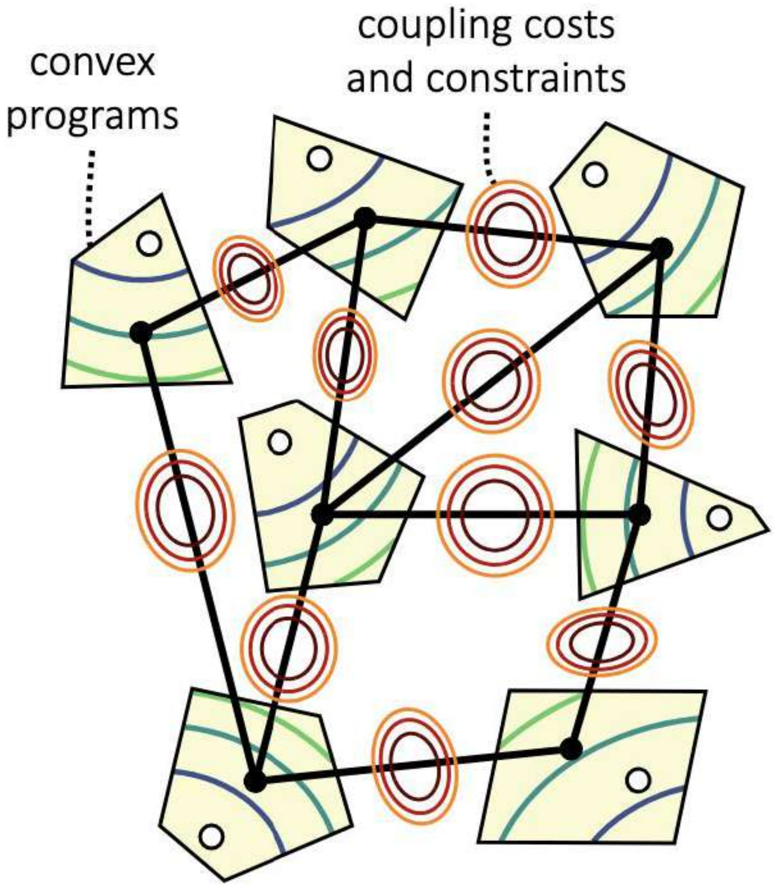
[1] Tang, Jingtao, and Hang Ma. "GHOST: Solving the Traveling Salesman Problem on Graphs of Convex Sets." Proceedings of the AAAI Conference on Artificial Intelligence. Vol. 40. No. 43. 2026.

Graphs of Convex Sets (GCS) [1]



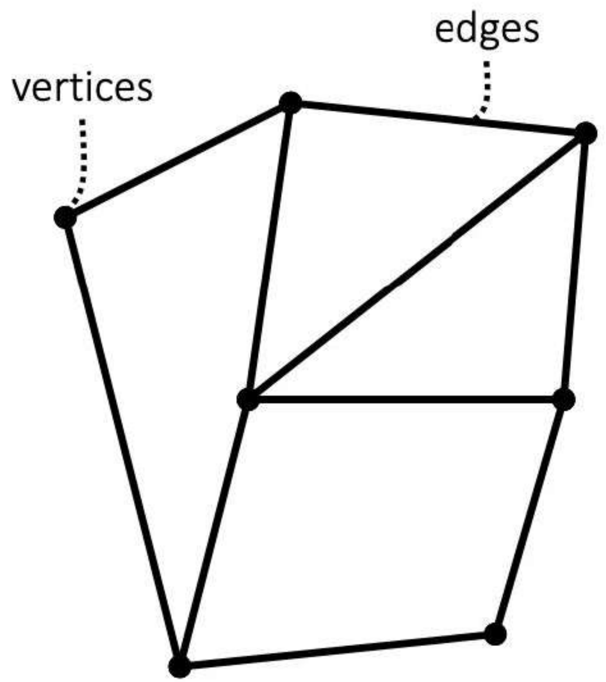
Standard graph with discrete vertices and edges.

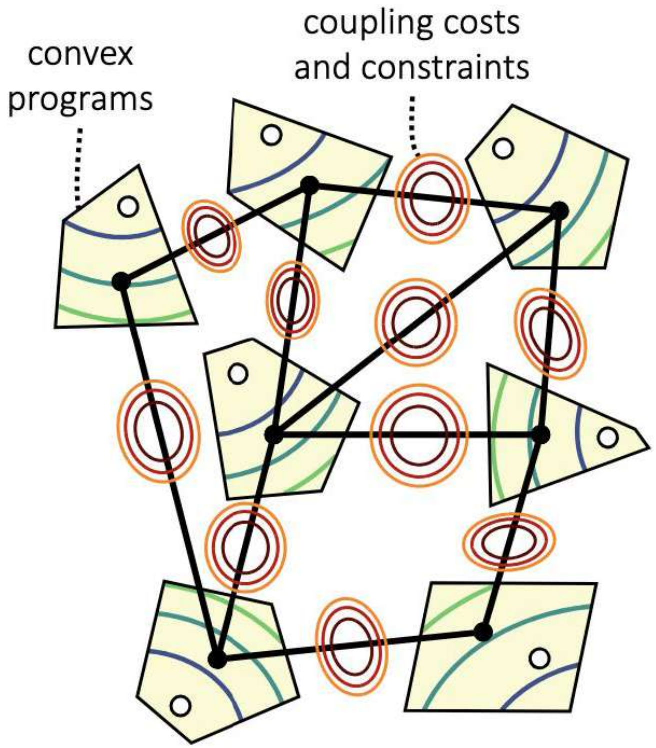


GCS where vertices and edges are paired with convex programs (i.e., convex costs and constraints).

[1] Marcucci, Tobia. *Graphs of convex sets with applications to optimal control and motion planning*. Diss. Massachusetts Institute of Technology, 2024.

Graphs Optimization Problems in GCS [1]





Graph Optimization Problem:

1. Discrete edge variables
2. Continuous flow variables (*)

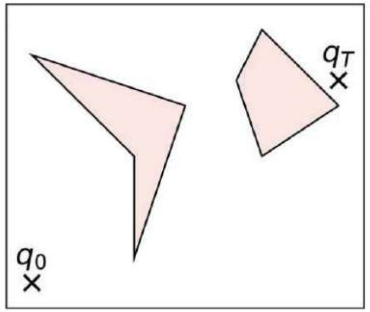
GCS Optimization Problems:

1. Discrete edge variables
2. Continuous flow variables (*)
3. Continuous point(s)-in-set variables

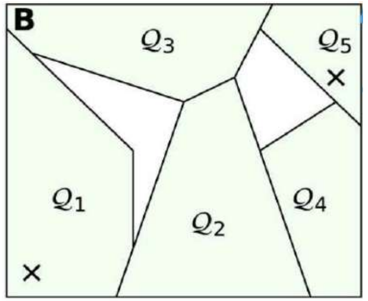
[1] Marcucci, Tobia. *Graphs of convex sets with applications to optimal control and motion planning*. Diss. Massachusetts Institute of Technology, 2024.

Motion Planning by Solving GCS Problems

Collision-Free Configuration Space (C-Space)

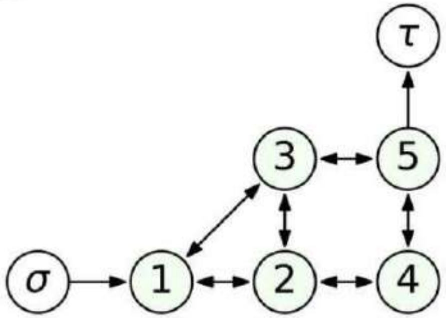


C-Space



Collision-free C-Space of Convex Sets [1,2]

build an edge if two sets intersect

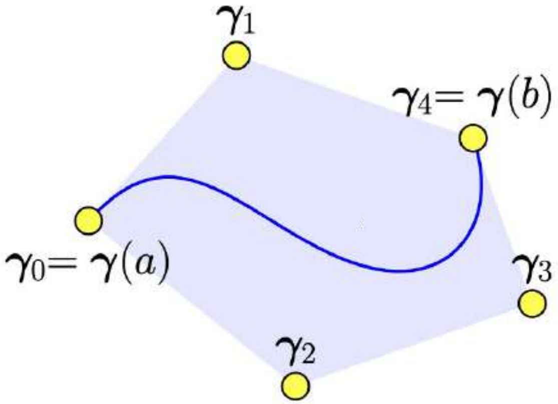


Corresponding GCS

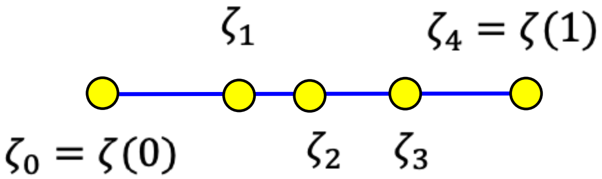
Motion Trajectory Representation

Foreach Convex Set

Space Bézier curve $\gamma: [0,1] \to \mathbb{R}^n$



Time "warping" Bézier curve $\zeta: [0,1] \to \mathbb{R}$



A C^{n-2}-continuous Bézier Curve

$\tau = \gamma \circ \zeta^{-1}: \mathbb{R} \to \mathbb{R}^n$

C^0: continous in pos.
C^1: continous in vel.
C^2: continous in acc.
C^3: continous in force
...

[1] Dai, Hongkai, et al. "Certified polyhedral decompositions of collision-free configuration space." *The International Journal of Robotics Research* 43.9 (2024): 1322-1341.
[2] Werner, Peter, et al. "Approximating robot configuration spaces with few convex sets using clique covers of visibility graphs." ICRA 2024

Traveling Salesman Problem (TSP) in GCS

Mixed-Integer Bilinear Program for TSP in GCS [1]

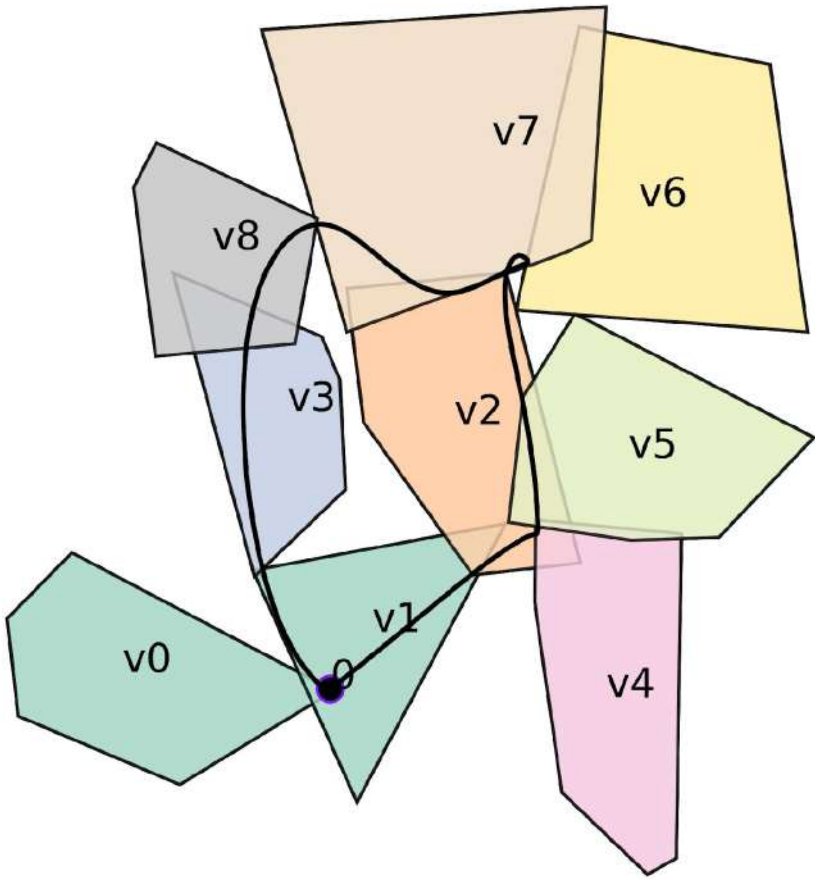
$$\min_{\mathbf{x},\mathbf{y}} \sum_{v \in V} y_v f_v(\mathbf{x}_v) + \sum_{e=(u,v) \in E} y_e f_e(\mathbf{x}_u, \mathbf{x}_v) \quad (1)$$

$$\text{s.t.} \quad \mathbf{y} \in \mathcal{Y} \subseteq \{0,1\}^{V \cup E}, \quad (2)$$

$$y_v \mathbf{x}_v \in y_v \mathcal{X}_v, \quad \forall v \in V, \quad (3)$$

$$y_e(\mathbf{x}_u, \mathbf{x}_v) \in y_e \mathcal{X}_e, \quad \forall e = (u,v) \in E \quad (4)$$

Highly Complex to Solve!
(both time & memory)



- **Decision Variables**

  x: Continuous Point(s)-in-Set Variables (control points)
  y: Discrete Edge Variables

- **Constraints**

  (2) Each vertex is visited at least once
  (3) Each points-in-set is bounded by the vertex convex set
  (4) Continuity between adjacent Bezier curve segments

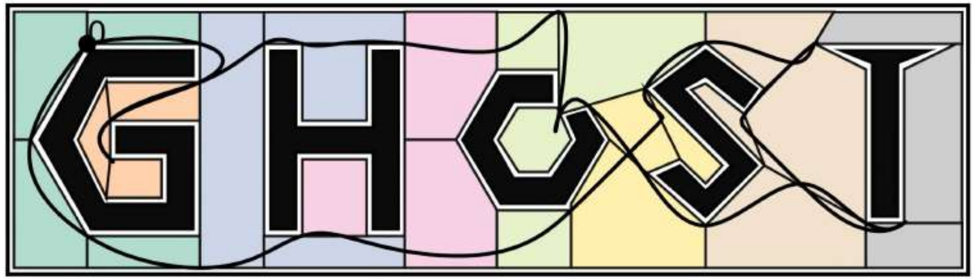
- **Objective:** (1) any convex function (e.g., trajectory cost)

[1] Marcucci, Tobia. *Graphs of convex sets with applications to optimal control and motion planning*. Diss. Massachusetts Institute of Technology, 2024.

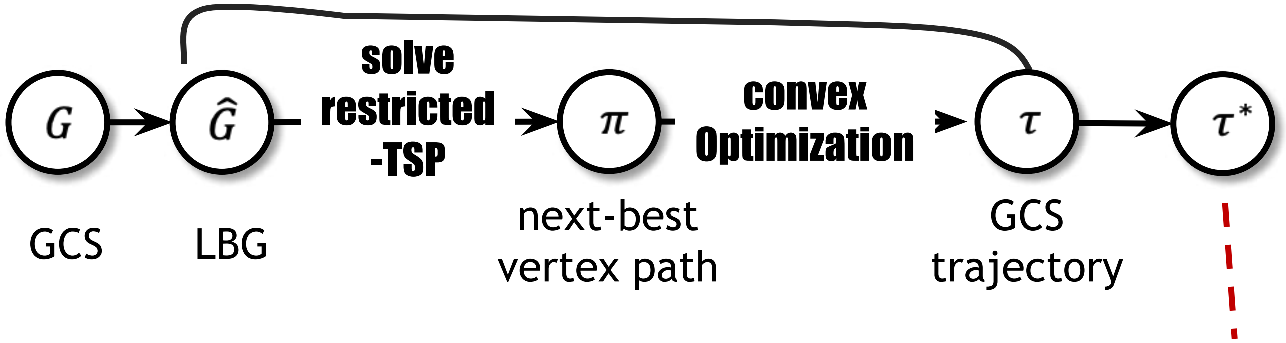
GCS-Hierarchical Search Framework (GHOST)

Separately The Optimizations of the Discrete Variables & Continuous Variables

- ❑ Which vertex path π (discrete variables $\mathbf{y}$) results the optimal GCS trajectory with minimal cost?
  - ✓ **Triplet Lower-Bound Graph (LBG)** [1]: a discrete graph $\hat{G}$ of "triplets" without the convex sets in the original GCS G, where trajectory cost $c_{\hat{G}}(\pi) \leq c_G(\tau|\pi)$ for any path π in G.
  - ✓ **Restricted-TSP (RTSP)** [2]: Each instance tuple $(\hat{G}, E^+, E^-)$ indicates inclusive edge set E^+ and exclusive edge set E^- for the TSP tour on $\hat{G}$



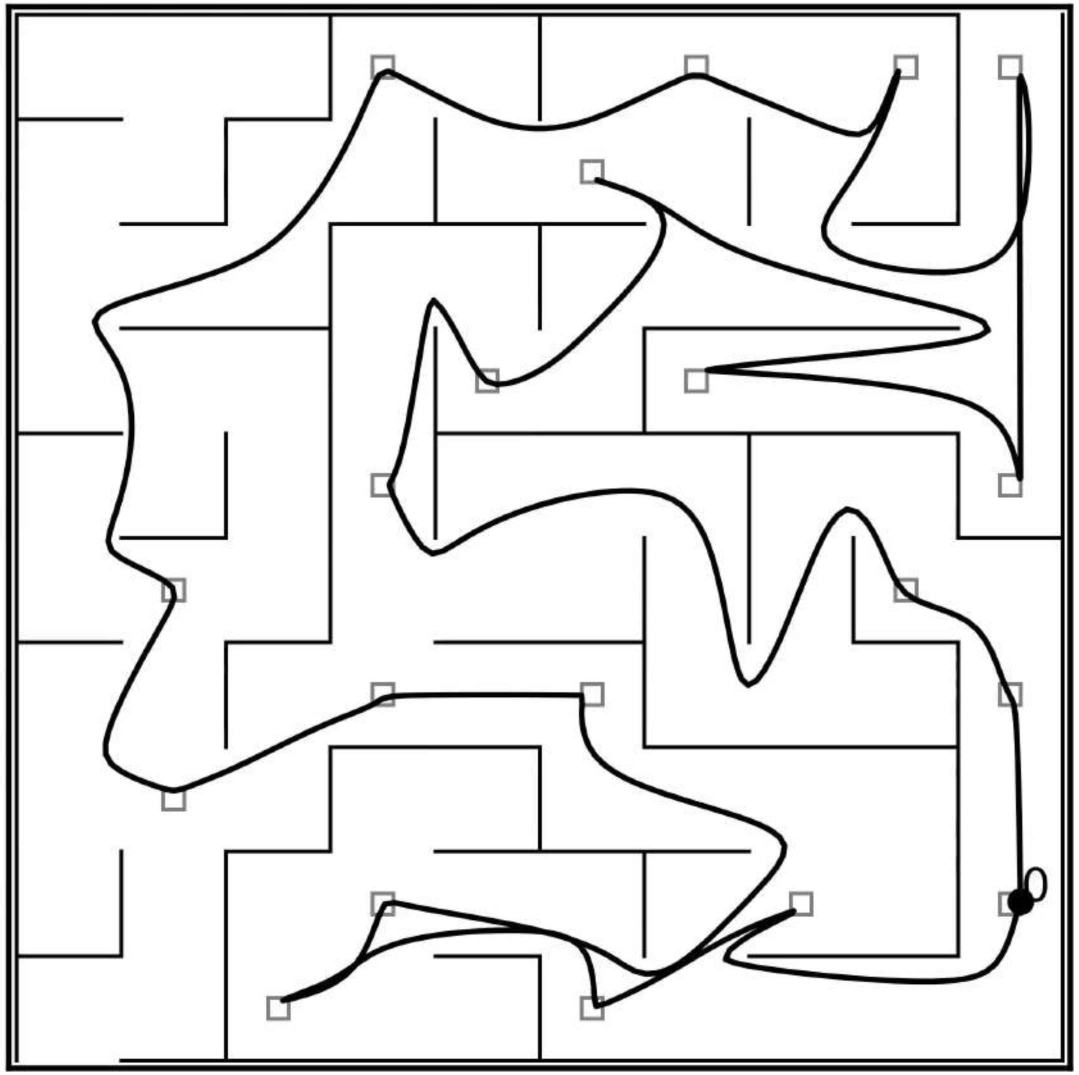
- ❑ What is the optimal trajectory τ (continuous variables $\mathbf{x}$) "*conditioned*" on a vertex path π?
  - ✓ **Convex Restriction:** fix (condition) the discrete variables in the MICP to π, then solve the resulting convex program (can be solved efficiently)



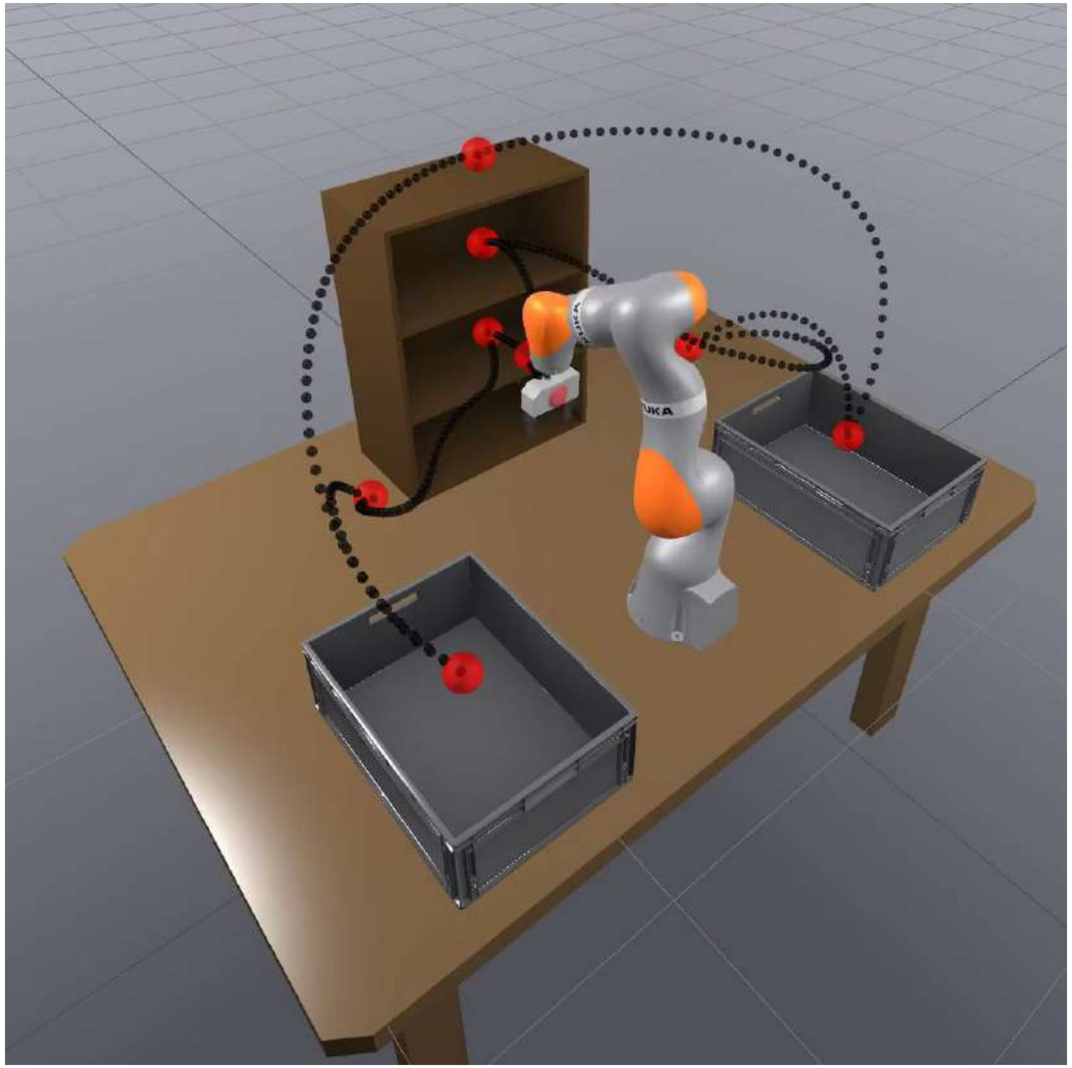
Terminate once the lower-bound cost $c_{\hat{G}}(\pi) \geq c_G(\tau^*|\pi^*)$ for the current-best trajectory τ^* and its conditioning path π^*

[1] Natarajan et al. "INSATxGCS (IxG): Implicit Graph Search for Planning on Graphs of Convex Sets." RSS 2024.
[2] Lawler, Eugene L. "A procedure for computing the k best solutions to discrete optimization problems and its application to the shortest path problem." *Management science* 18.7 (1972)

Robotics Applications of TSP in GCS



Coverage/Inspection Planning



Task and Motion Planning (7-DoF)